Parallel Running System of Three Oscillators Coupled Through a Six-Port Magic Junction

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Abstract —This paper treats parallel running systems of three oscillators symmetrically coupled to one another through a six-port magic junction and closely examines two systems for power combining. One system delivers a combined power to the coaxial arm, the other to a circular waveguide in the form of a circular polarization. The former is marked by its easy adjustability and simple structure and the latter by its output form and the electrical switch of the direction of rotation of circular polarization. Experimental verification for the two power-combining systems is presented.

I. Introduction

POWER-COMBINING techniques for microwave oscillators have been intensively studied since microwave solid-state devices, such as Gunn and IMPATT diodes, were invented. The techniques may be classified broadly into two categories [1]: 1) multiple-device oscillators and 2) power-combining systems of two or more oscillators.

The former originated in a five-IMPATT coaxial-type oscillator in which the IMPATT diodes were coupled through a nonresonant circuit by Rucker [2] and a 12-IMPATT waveguide-type oscillator by Kurokawa and Magalhaes [3]. This technique has attracted much interest for its compact structure, the relative ease with which the oscillation can be adjusted, and the low loss in coupling. Consequently, various multiple-device oscillators with IMPATT's, Gunn diodes, and GaAs FET's mounted in a rectangular or a cylindrical resonant cavity have been developed [4]–[7]. These oscillators, however, are generally limited in the number of devices by the dimensions of the device-mounting structure and the cavity and to the suppression of unwanted oscillation modes.

Pioneering studies of power-combining systems involving two or more oscillators include the eight-IMPATT oscillators by Fukui [8] and the 32-tunnel-diode oscillators by Mizushina [9]. Thereafter parallel running of multiple oscillators has also been actively investigated with theoretical interest in a mutual synchronization of oscillators as a typical nonlinear phenomenon [10]–[15]. The well-known combining structures are a 2^N-oscillator system coupled through 2^N-1 3 dB hybrids such as a magic T [8], [9], and an N-oscillator system serially connected by $N-1$ directional couplers with coupling factors of 3 dB, 4.78 dB, and so forth [12]. Although these parallel running systems have the drawbacks of being large in size, having oscillations that are relatively complicated to adjust, and having a combining efficiency that deteriorates with the multiplication of the number of oscillators, the combining techniques are useful for constructing an MIC multiple-oscillator system, and an ultra-multiple-device oscillator system with a hundred or more active devices by the use of several multiple-device oscillators, and for combining the powers from existing oscillators.

If an easily adjustable combiner that is able to couple more oscillators per stage is employed, the above mentioned disadvantages of a parallel running system are considerably eased. Mizushina *et al.* have proposed a simpler multiple-oscillator system in which a basic element is constructed of three (multiple-device) oscillators coupled through a short-slot 3 dB coupler [15], [16]. This system, however, appears to be difficult to analyze and complicated to adjust, because each coupling between the oscillators is unbalanced and a synchronized state is optimized by adjusting the frequency of each oscillator in operation.

This paper treats parallel running systems of three oscillators symmetrically coupled through a six-port magic junction [17]. The treatment is prompted by a practical interest in power combining and a theoretical interest in the phenomenon of mutual synchronization of three oscillators, and it closely examines two definite power combining systems. One system delivers a combined power to the coaxial arm; the other a combined power to the circular waveguide in a mode with circular polarization. These systems are easy to analyze and adjust from the two reasons: coupling between the three oscillators is balanced and a synchronized state can be adjusted independently of the oscillators as in the two-oscillator system constructed with a magic T. In Section II we first outline the underlying principles of the three-oscillator system, and point out that the system can provide the two above-mentioned different types of combined powers according to the circuit conditions. Section III is devoted to a derivation of the synchronized steady-state solutions of the oscillator system. In Section IV, we obtain variational equations

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necessary for dealing with the stability of the steady-state solutions. In Section V, we consider two systems with a TEM output and a circularly polarized output concretely, and derive the stability conditions of each synchronization mode in the two systems. Finally, in Section VI, two different experimental systems for power combining are constructed and tested to confirm theoretical results.

II. Construction of Parallel Running System

A six-port magic junction (Fig. 1) is employed as a power combiner. Numbering the ports symmetrically locating three reference planes, $t_1 \sim t_3$, in the three rectangular waveguides and designating the reference planes in the coaxial arm and the circular arm for two polarizations as t_4, t_5, and t_6, respectively, as shown in Fig. 1, we can exhibit the scattering matrix in the form [17]

$$[S]=\begin{bmatrix} 0 & 0 & 0 & 1/\sqrt{3} & \sqrt{2/3} & 0 \\ 0 & 0 & 0 & 1/\sqrt{3} & -1/\sqrt{6} & 1/\sqrt{2} \\ 0 & 0 & 0 & 1/\sqrt{3} & -1/\sqrt{6} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 & 0 & 0 \\ \sqrt{2/3} & -1/\sqrt{6} & -1/\sqrt{6} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

Substituting (1) into $[b_1, b_2, \cdots, b_6]^t = [S][a_1, a_2, \cdots, a_6]^t$ and modifying the resultant equation, we obtain

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ (b_5 + jb_6)/\sqrt{2} \\ (b_5 - jb_6)/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & e^{j2\pi/3} & e^{-j2\pi/3} \\ 0 & 0 & 0 & 1 & e^{-j2\pi/3} & e^{j2\pi/3} \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & e^{j2\pi/3} & e^{-j2\pi/3} & 0 & 0 & 0 \\ 1 & e^{-j2\pi/3} & e^{j2\pi/3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ (a_5 - ja_6)/\sqrt{2} \\ (a_5 + ja_6)/\sqrt{2} \end{bmatrix} \quad (2)$$

where a_i and b_i $(i = 1, 2, \cdots, 6)$ are the incident and scattered power waves of the ith port, respectively. Moreover, notice that right- and left-hand circularly polarized waves are selected as the variables in the circular waveguide. Equation (2) indicates that equal-amplitude waves incident in the three rectangular waveguides completely combine into a TEM wave in the coaxial arm or into either of two circularly polarized waves in the circular arm depending on whether their phases are in-phase or three-phase.

We now consider the power combining system with the three oscillators symmetrically connected to the three rectangular waveguides of the above six-port magic junction as shown in Fig. 2. If both circuits attached to the coaxial and the circular arms are adjusted to stabilize an in-phase synchronization, the powers from the three oscillators completely combine and emerge from the coaxial arm. Accordingly, since no power is delivered to the circuit of the circular arm, the circuit seems to be unnecessary. But actually it plays the important role of holding the in-phase synchronization, as will be shown later.

If, on the other hand, a three-phase synchronization is stabilized, a combined wave in a right- or left-hand circular polarization is produced in the circular waveguide. In this case, it does not necessarily follow that we can obtain a circular polarization with a desired direction of rotation, because the two three-phase synchronizations are degenerate for a reciprocal circular waveguide circuit with rotational symmetry. To separate the two modes, the degeneracy is to be resolved. This can be achieved by, for example, connecting a nonreciprocal circuit to the circular waveguide, as will be discussed in Section V.

III. Steady-State Solutions

In Fig. 2, $[Y]$ indicates an admittance matrix looking toward the coupling circuit inclusive of variable reflectors (or loads) at the symmetrically located reference planes of three oscillators. On the assumption that each oscillator has the same parameters except for the free-running frequency, let us represent the admittance of the pth oscillator as

$$Y_{Gp} = G_e(V_p) + jQ_e(\omega - \omega_{0p})/\omega_0, \qquad p = 1, 2, 3 \quad (3)$$

where $G_e(V_p)$ is the negative conductance, V_p the amplitude of the oscillation voltage, ω_{0p} the free-running angular frequency, Q_e the external Q, and $\omega_0 = (\omega_{01} + \omega_{02} + \omega_{03})/3$.

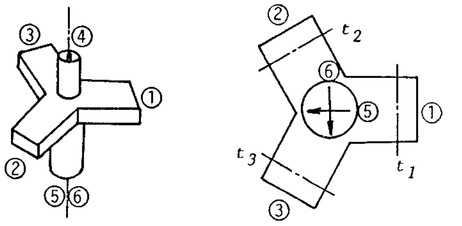
Now, considering the condition whereby the total admittance is equal to zero in Fig. 2, we obtain the following condition necessary for synchronized oscillation:

$$([Y]+[Y_G])\boldsymbol{v} = 0 \quad (4)$$

where $\boldsymbol{v}$ is the voltage vector with its components given by the RF voltages at respective reference planes, and

$$[Y_G] = \text{diag}\,[Y_{G1}, Y_{G2}, Y_{G3}]. \quad (5)$$

Next, let it be assumed that all the ω_{0p} are of the same



(a) (b)

Fig. 1. (a) Schematic configuration of a six-port magic junction employed as a combiner. (b) Terminals in the circular guide for two polarizations.

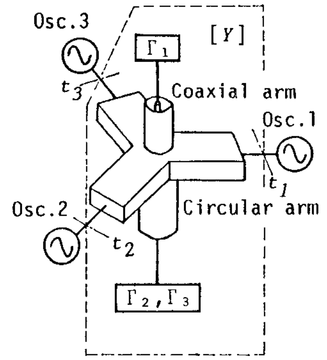


Fig. 2. Block diagram of a parallel running system of three oscillators coupled through a six-port magic junction.

value. Then, all the oscillators see the same admittance owing to the symmetry of the coupling circuit and the identity of the three oscillators, and we have $V_1 = V_2 = V_3 (\equiv V)$, i.e., $Y_{G1} = Y_{G2} = Y_{G3} (\equiv Y_G)$. As a result, (4) becomes the eigenvalue problem of $[Y]$.

From the third-order rotational symmetry, the eigenvectors of $[Y]$ are given by

$$\boldsymbol{u}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}, \qquad \boldsymbol{u}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\e^{-j2\pi/3}\\e^{j2\pi/3}\end{bmatrix},$$

$$\boldsymbol{u}_3 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\e^{j2\pi/3}\\e^{-j2\pi/3}\end{bmatrix} \tag{6}$$

and the corresponding eigenvalues by

$$y_i = \frac{1-\Gamma_i}{1+\Gamma_i}, \qquad i = 1,2,3 \tag{7}$$

where Γ_1 represents the voltage reflection coefficient of the variable reflector in the coaxial arm (its phase is referred to at the three symmetrical reference planes), and Γ_2 and Γ_3 those for right- and left-hand circular polarizations in the circular waveguide, respectively.

Consequently, we find that this system falls into one of the following three synchronized steady-state solutions:

1) In-phase synchronization:

$$Y_G + y_1 = 0, \qquad \boldsymbol{v} = \sqrt{3}\,V\boldsymbol{u}_1. \tag{8}$$

This mode of synchronization can combine the three powers into a TEM wave in the coaxial arm, as mentioned above.

2) Three-phase synchronizations:

$$Y_G + y_2 = 0, \qquad \boldsymbol{v} = \sqrt{3}\,V\boldsymbol{u}_2 \tag{9}$$

$$Y_G + y_3 = 0, \qquad \boldsymbol{v} = \sqrt{3}\,V\boldsymbol{u}_3. \tag{10}$$

These second and third modes produce a right- and a left-hand circularly polarized output into the circular waveguide, respectively.

Moreover, it is found from (8)–(10) that, in every case, each oscillator behaves as if it were terminated in a load with the same admittance as the corresponding eigenvalue (eigenadmittance).

IV. Variational Equations

In this section, we derive variational equations fundamental to treating the stability of the above synchronized steady-state solutions. Now, let it be assumed that the amplitude and the phase of the oscillation voltage of the pth oscillator deviate by slight values of δV_p and $\delta\theta_p$, respectively, from the ith steady-state solution for one reason or another. Then we can rewrite the voltage vector with the first-order approximation as follows:

$$\boldsymbol{v} = \boldsymbol{v}_{is} + \delta \boldsymbol{V}_i + jV\delta\boldsymbol{\theta}_i \tag{11}$$

where $\boldsymbol{v}_{is} = \sqrt{3}\,V\boldsymbol{u}_i$ and

$$\delta\boldsymbol{V}_i = \mathrm{diag}[e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}]\delta\boldsymbol{V} \tag{12}$$

$$\delta\boldsymbol{\theta}_i = \mathrm{diag}[e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}]\delta\boldsymbol{\theta}. \tag{13}$$

To complete the list of definitions we have θ_p ($p = 1,2,3$) = oscillation phase of the pth oscillator in the ith synchronism:

$$\delta\boldsymbol{V} = \begin{bmatrix}\delta V_1\\ \delta V_2\\ \delta V_3\end{bmatrix} \qquad \delta\boldsymbol{\theta} = \begin{bmatrix}\delta\theta_1\\ \delta\theta_2\\ \delta\theta_3\end{bmatrix}.$$

With consideration of the fact that the term $j\omega$ in an admittance operated on v_p indicates dv_p/dt in a dynamical sense [18], substituting (11) into (4) gives

$$\sum_{r=1}^{3} Y_{pr}v_r + \Bigg\{ G_e(V + \delta V_p) + j2Q_e(\omega - \omega_0)/\omega_0 + j\frac{2Q_e}{\omega_0}\left(\frac{d\,\delta\theta_p}{dt} - j\frac{1}{V+\delta V_p}\frac{d\,\delta V_p}{dt}\right)\Bigg\} v_p = 0, p = 1,2,3 \tag{14}$$

where it is assumed that the coupling circuit is free of frequency dependence. To a first-order approximation with

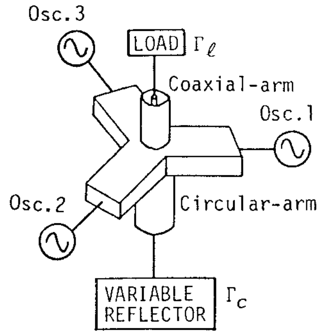


Fig. 3. Block diagram of a power-combining system delivering the sum of three outputs to a load in the coaxial arm

respect to small variations, the above equation can be rewritten in the matrix form

$$([Y]+Y_G[U])(\boldsymbol{v}_{is}+\delta \boldsymbol{V}_i+jV\delta\boldsymbol{\theta}_i)+s\delta\boldsymbol{V}_i + d\delta\boldsymbol{V}_i/d\tau + jVd\delta\boldsymbol{\theta}_i/d\tau = 0 \quad (15)$$

where $s=(dG_e/dV)_VV$, $[U]$ is the unit matrix, and $\tau = (\omega_0/2Q_e)t$.

Substituting the ith steady-state equation,

$$([Y]+Y_G[U])\boldsymbol{v}_{is}=0 \quad (16)$$

into (15) and using the spectral representation of $[Y]$, we have

$$\sum_{k=1}^{3}\left[\tilde{\boldsymbol{u}}_k\cdot\{(y_k+Y_G)(\delta\boldsymbol{V}_i+jV\delta\boldsymbol{\theta}_i)+s\delta\boldsymbol{V}_i + d\delta\boldsymbol{V}_i/d\tau + jVd\delta\boldsymbol{\theta}_i/d\tau\}\right]\boldsymbol{u}_k=0 \quad (17)$$

where $\tilde{\boldsymbol{u}}_k$ is the adjoint vector of $\boldsymbol{u}_k$. Furthermore, substituting the admittance expression in the ith steady-state,

$$Y_G+y_i=0 \quad (18)$$

into (17) and putting all the coefficients of $\boldsymbol{u}_k$ in the resultant equation into zero from the orthonormal conditions between $\boldsymbol{u}_k$'s, we can derive the following variational equations around the ith steady-state solution:

$$\tilde{\boldsymbol{u}}_k\cdot\{(y_k-y_i)(\delta\boldsymbol{V}_i+jV\delta\boldsymbol{\theta}_i)+s\delta\boldsymbol{V}_i + d\delta\boldsymbol{V}_i/d\tau + jVd\delta\boldsymbol{\theta}_i/d\tau\}=0, \qquad k=1,2,3. \quad (19)$$

V. Condition for Stability

A. System with Combined Output Power to a Coaxial Load

We consider the power-combining system shown in Fig. 3, where the coaxial arm is terminated in a load with a reflection coefficient of Γ_ℓ and the circular arm in a variable reflector with a reflection coefficient of Γ_c regardless of the direction of polarization. Hence, in the following we should note that the three-phase synchronizations of this system are degenerate. After closely examining the stability conditions for each mode of synchronization, we will obtain the circuit conditions for complete power combining.

1) Stability Conditions for In-Phase Synchronization: In this case, since $\delta\boldsymbol{V}_1=\delta\boldsymbol{V}$ and $\delta\boldsymbol{\theta}_1=\delta\boldsymbol{\theta}$, from (19) we obtain

$$\frac{d(\tilde{\boldsymbol{u}}_1\cdot\delta\boldsymbol{V})}{d\tau}+s(\tilde{\boldsymbol{u}}_1\cdot\boldsymbol{V})+jV\frac{d(\tilde{\boldsymbol{u}}_1\cdot\delta\boldsymbol{\theta})}{d\tau}=0 \quad (20)$$

$$\tilde{\boldsymbol{u}}_k\cdot[d\delta\boldsymbol{V}/d\tau+s\delta\boldsymbol{V}+\mathrm{Re}(y_k-y_1)\delta\boldsymbol{V} - V\,\mathrm{Im}(y_k-y_1)\delta\boldsymbol{\theta}+j\{Vd\delta\boldsymbol{\theta}/d\tau + V\,\mathrm{Re}(y_k-y_1)\delta\boldsymbol{\theta}+\mathrm{Im}(y_k-y_1)\delta\boldsymbol{V}\}]=0, \qquad k=2,3. \quad (21)$$

Paying attention to the fact that $\boldsymbol{u}_1$, $\delta\boldsymbol{V}$, and $\delta\boldsymbol{\theta}$ are real vectors, we can divide (20) into the following two equations:

$$d(\tilde{\boldsymbol{u}}_1\cdot\delta\boldsymbol{V})/d\tau+s(\tilde{\boldsymbol{u}}_1\cdot\delta\boldsymbol{V})=0 \quad (22a)$$

$$d(\tilde{\boldsymbol{u}}_1\cdot\delta\boldsymbol{\theta})/d\tau=0. \quad (22b)$$

On the other hand, when $y_2=y_3$, i.e., in the present case, it is evident from (21) that the term in brackets is orthogonal to both $\boldsymbol{u}_2$ and $\boldsymbol{u}_3$, and hence is proportional to the real vector $\boldsymbol{u}_1$. Accordingly, we can see that the real and imaginary parts in the brackets are each proportional to $\boldsymbol{u}_1$. Thus,

$$d(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{V})/d\tau+\{s+\mathrm{Re}(y_k-y_1)\}(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{V}) - \mathrm{Im}(y_k-y_1)V(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{\theta})=0 \quad (23a)$$

$$\cdot Vd(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{\theta})/d\tau+\mathrm{Re}(y_k-y_1)V(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{\theta}) + \mathrm{Im}(y_k-y_1)(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{V})=0, \qquad k=2,3. \quad (23b)$$

For stable in-phase synchronization all $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, and $\boldsymbol{u}_3$ components of $\delta\boldsymbol{V}$ and $\delta\boldsymbol{\theta}$ have to decrease with time. For the $\boldsymbol{u}_1$ component of $\delta\boldsymbol{V}$, the inequality

$$s>0 \quad (24)$$

is required from (22a), while no condition for the $\boldsymbol{u}_1$ component of $\delta\boldsymbol{\theta}$ is required from (22b). Since an ordinary self-excited oscillator has $s\simeq 2$ [19], the above inequality is satisfied generally. For the $\boldsymbol{u}_2$ and $\boldsymbol{u}_3$ components of $\delta\boldsymbol{V}$ and $\delta\boldsymbol{\theta}$, two roots of the following characteristic equation derived from (23) must have negative real parts:

$$\begin{vmatrix} s+\mathrm{Re}(y_c-y_1)+\lambda & -\mathrm{Im}(y_c-y_1)\cdot V \\ \mathrm{Im}(y_c-y_1)/V & \mathrm{Re}(y_c-y_1)+\lambda \end{vmatrix}=0 \quad (25)$$

where

$$y_c\equiv y_2=y_3=(1-\Gamma_c)/(1+\Gamma_c). \quad (26)$$

To satisfy the above requirements, it is necessary and sufficient that

$$s+2\,\mathrm{Re}(y_c-y_1)>0 \quad (27a)$$

$$s\,\mathrm{Re}(y_c-y_1)+\{\mathrm{Re}(y_c-y_1)\}^2+\{\mathrm{Im}(y_c-y_1)\}^2>0 \quad (27b)$$

hold.

2) Stability Conditions for Three-Phase Synchronizations: First, we deal with the second mode ($i=2$) given by (9).

Substituting $i = 2$ and (26) into (19) yields

$$\tilde{\boldsymbol{u}}_1 \cdot \{(y_1 - y_c)(\delta V_2 + jV\delta\boldsymbol{\theta}_2) + s\delta V_2 + d\delta V_2/d\tau + jVd\delta\boldsymbol{\theta}_2/d\tau\} = 0 \quad (28)$$

$$\tilde{\boldsymbol{u}}_2 \cdot (s\delta V_2 + d\delta V_2/d\tau + jVd\delta\boldsymbol{\theta}_2/d\tau) = 0 \quad (29)$$

$$\tilde{\boldsymbol{u}}_3 \cdot (s\delta V_2 + d\delta V_2/d\tau + jVd\delta\boldsymbol{\theta}_2/d\tau) = 0. \quad (30)$$

The Hermitian inner product of each eigenvector and δV_2 or $\delta\boldsymbol{\theta}_2$ can be rewritten as

$$\tilde{\boldsymbol{u}}_1 \cdot (\delta V_2, \delta\boldsymbol{\theta}_2) = (\tilde{\boldsymbol{u}}_{3R} - j\tilde{\boldsymbol{u}}_{3I}) \cdot (\delta V, \delta\boldsymbol{\theta}) \quad (31a)$$

$$\tilde{\boldsymbol{u}}_2 \cdot (\delta V_2, \delta\boldsymbol{\theta}_2) = \tilde{\boldsymbol{u}}_1 \cdot (\delta V, \delta\boldsymbol{\theta}) \quad (31b)$$

$$\tilde{\boldsymbol{u}}_3 \cdot (\delta V_2, \delta\boldsymbol{\theta}_2) = (\tilde{\boldsymbol{u}}_{3R} + j\tilde{\boldsymbol{u}}_{3I}) \cdot (\delta V, \delta\boldsymbol{\theta}) \quad (31c)$$

where $\boldsymbol{u}_3$ is replaced in the form

$$\boldsymbol{u}_3 = \boldsymbol{u}_{3R} + j\boldsymbol{u}_{3I} \quad (32)$$

by the use of real vectors $\boldsymbol{u}_{3R}$ and $\boldsymbol{u}_{3I}$. Since, as can be seen from (31b), $\tilde{\boldsymbol{u}}_2 \cdot (\delta V_2, \delta\boldsymbol{\theta}_2)$ is a real number, (29) becomes the same as (22). Therefore, no condition but (24) is derived from (31b). Substituting (31a) and (31c) into (28) and (30), respectively, and equating the real and imaginary parts separately, we have

$$d\delta V_R/d\tau + Vd\delta\theta_I/d\tau + s\delta V_R + \mathrm{Re}(y_1 - y_c)(\delta V_R + V\delta\theta_I) + \mathrm{Im}(y_1 - y_c)(\delta V_I - V\delta\theta_R) = 0 \quad (33a)$$

$$d\delta V_I/d\tau - Vd\delta\theta_R/d\tau + s\delta V_I + \mathrm{Re}(y_1 - y_c)(\delta V_I - V\delta\theta_R) - \mathrm{Im}(y_1 - y_c)(\delta V_R + V\delta\theta_I) = 0 \quad (33b)$$

$$d\delta V_R/d\tau - Vd\delta\theta_I/d\tau + s\delta V_R = 0 \quad (34a)$$

$$d\delta V_I/d\tau + Vd\delta\theta_R/d\tau + s\delta V_I = 0 \quad (34b)$$

where

$$\delta V_{R,I} = \tilde{\boldsymbol{u}}_{3R,I} \cdot \delta \boldsymbol{V} \qquad \delta\theta_{R,I} = \tilde{\boldsymbol{u}}_{3R,I} \cdot \delta\boldsymbol{\theta}. \quad (35)$$

Equations (33) and (34) give the characteristic equation

$$\lambda^4 + p_3\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \quad (36)$$

where

$$p_0 = s^2(y_R^2 + y_I^2)/4 \quad (37a)$$

$$p_1 = s(y_R^2 + y_I^2 + sy_R) \quad (37b)$$

$$p_2 = y_R^2 + y_I^2 + 3sy_R + s^2 \quad (37c)$$

$$p_3 = 2(y_R + s) \quad (37d)$$

with the replacements of

$$y_R = \mathrm{Re}(y_1 - y_c) \qquad y_I = \mathrm{Im}(y_1 - y_c). \quad (38)$$

Applying the Routh–Hurwitz criterion to (36) brings the following necessary and sufficient conditions that all the roots of (36) have negative real parts for $p_0 > 0$ [20]:

$$p_1 > 0 \quad (39a)$$

$$\begin{vmatrix} p_1 & p_0 \\ p_3 & p_2 \end{vmatrix} > 0 \quad (39b)$$

$$\begin{vmatrix} p_1 & p_0 & 0 \\ p_3 & p_2 & p_1 \\ 0 & 1 & p_3 \end{vmatrix} > 0. \quad (39c)$$

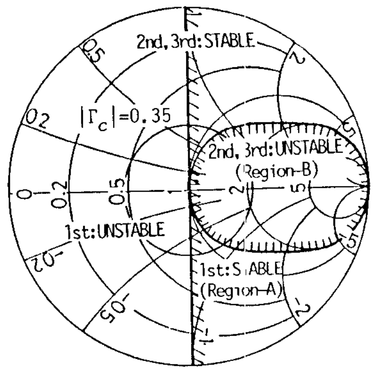


Fig. 4. Stability region of the in-phase synchronization (region A) and instability region of the other synchronizations (region B) on a Smith admittance chart for a circular arm circuit (Γ_c) of a power-combining system delivering output power to a matched load ($\Gamma_l = 0$).

If

$$\{\mathrm{Im}(y_1 - y_c)\}^2\{s + 2\,\mathrm{Re}(y_1 - y_c)\} + 2\,\mathrm{Re}(y_1 - y_c)\{s + \mathrm{Re}(y_1 - y_c)\}^2 > 0 \quad (40)$$

all the inequalities of (39) are satisfied. Accordingly, (40) is the stability condition for the second mode.

To the third mode also (40) applies, because we can have the same discussion by the interchange of subscripts 2 and 3.

3) Circuit Condition for Power Combining: Let us examine actual circuit conditions for complete power combining by means of the above stability conditions, (27) and (40). If a matched load is connected to the coaxial arm, we have

$$y_\ell = 1 \quad \text{or} \quad \Gamma_\ell = 0. \quad (41)$$

Moreover, let it be assumed that

$$s = 2. \quad (42)$$

Then, a region where y_c (Γ_c) satisfies (27), in other words a stability region of the power-combining synchronization for a circular arm circuit, can be drawn as region A on the Smith admittance chart in Fig. 4. On the other hand, as a region where (40) is unsatisfied, i.e., an instability region of the other modes, we obtain region B in the same figure. Thus, it is found that by fixation of Γ_c in the common region between both regions (i.e., region B), only the first mode (in-phase mode) can be realized with the desired stability; hence a completely combined power can be delivered to the matched load in the coaxial arm.

B. System with Combined Output Power in a Circularly Polarized Wave

Fig. 5(a) exhibits a specific circuit configuration to be discussed. A variable reflector is connected to the coaxial arm, and a Faraday rotator, a centered circular aperture in a transverse metallic plate, and a (symmetrical) load in that order are connected to the circular arm. In the figure,

Γ_1 represents a reflection coefficient of the variable termination in the coaxial arm, while Γ_2 and Γ_3 represent those of the circular waveguide circuits for right- and left-hand circular polarizations, respectively. The magnitudes of Γ_2 and Γ_3 are equally adjusted by the size of the circular aperture, but their phase angles differently by the exciting current of the Faraday rotator. Since the Faraday rotator gives different phase shifts to a right- and a left-hand circular polarization, the two three-phase synchronizations of this system become nondegenerate. Therefore, we can distinguishably obtain one of the two circularly polarized outputs. In addition, it is also possible electrically to switch its direction of rotation since the phase angles of Γ_2 and Γ_3 are exchanged for each other by inversion of the exciting current of the Faraday rotator.

Now, let us analyze the stability conditions for the ith synchronization mode ($i = 1, 2, 3$). The present system is a little more complicated than the first mode system because of the nondegeneracy of the three-phase synchronization modes. Let it be assumed that integers i, j, and k in that order are equal to 1, 2, and 3 in the cyclic order; for example if $i = 3$, we have that $j = 1$ and $k = 2$. Then, by inspecting (6), (12), (13), and (35), we can obtain

$$\tilde{\boldsymbol{u}}_i \cdot \delta \boldsymbol{V}_i = \tilde{\boldsymbol{u}}_1 \cdot \delta \boldsymbol{V} \qquad \tilde{\boldsymbol{u}}_i \cdot \delta \boldsymbol{\theta}_i = \tilde{\boldsymbol{u}}_1 \cdot \delta \boldsymbol{\theta} \tag{43}$$

$$\tilde{\boldsymbol{u}}_j \cdot \delta \boldsymbol{V}_i = \delta V_R + j\delta V_I \qquad \tilde{\boldsymbol{u}}_j \cdot \delta \boldsymbol{\theta}_i = \delta \theta_R + j\delta \theta_I \tag{44}$$

$$\tilde{\boldsymbol{u}}_k \cdot \delta \boldsymbol{V}_i = \delta V_R - j\delta V_I \qquad \tilde{\boldsymbol{u}}_k \cdot \delta \boldsymbol{\theta}_i = \delta \theta_R - j\delta \theta_I. \tag{45}$$

Substituting (43), (44), and (45) into (19), we obtain the same equations as (22) and the following equations:

$$d\delta V_R/d\tau - Vd\delta\theta_I/d\tau + s\delta V_R + \mathrm{Re}(y_j - y_i)(\delta V_R - V\delta\theta_I) - \mathrm{Im}(y_j - y_i)(\delta V_I + V\delta\theta_R) = 0 \tag{46a}$$

$$d\delta V_I/d\tau + Vd\delta\theta_R/d\tau + s\delta V_I + \mathrm{Re}(y_j - y_i)(\delta V_I + V\delta\theta_R) + \mathrm{Im}(y_j - y_i)(\delta V_R - V\delta\theta_I) = 0 \tag{46b}$$

$$d\delta V_R/d\tau + Vd\delta\theta_I/d\tau + s\delta V_R + \mathrm{Re}(y_k - y_i)(\delta V_R + V\delta\theta_I) + \mathrm{Im}(y_k - y_i)(\delta V_I - V\delta\theta_R) = 0 \tag{47a}$$

$$d\delta V_I/d\tau - Vd\delta\theta_R/d\tau + s\delta V_I + \mathrm{Re}(y_k - y_i)(\delta V_I - V\delta\theta_R) - \mathrm{Im}(y_k - y_i)(\delta V_R + V\delta\theta_I) = 0. \tag{47b}$$

Actual stability conditions for this case are derived from only (46) and (47) in the same manner as before. However, a characteristic equation derived from the four equations, as can be forecast from the difference between (34) and (47), becomes more complicated than (36). For this reason, it is difficult to obtain analytically stability conditions in the form of an inequality such as (27) and (40). Fortunately, in the above discussion, we see that the main parts of a stability and an instability region are located where the imaginary part of the eigenadmittance, $y_{i,j,k}$, vanishes. So, let us examine the following particular case first:

$$\mathrm{Im}\, y_i = \mathrm{Im}\, y_j = \mathrm{Im}\, y_k = 0. \tag{48}$$

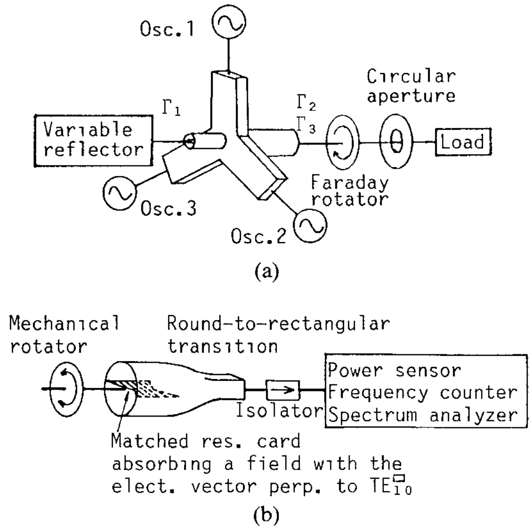


Fig. 5. Block diagrams of (a) an oscillator system directly combining three output powers into a circularly polarized wave and of (b) its load composed for measurements of output power in various directions of polarization.

Then the characteristic equation reduces to the second-order equation

$$2\lambda^2 + 2(s + y_{jiR} + y_{kiR})\lambda + s(y_{jiR} + y_{kiR}) + 2y_{jiR}y_{kiR} = 0 \tag{49}$$

where

$$y_{jiR} = \mathrm{Re}(y_j - y_i) \qquad y_{kiR} = \mathrm{Re}(y_k - y_i). \tag{50}$$

This equation derives the two conditions necessary and sufficient for stable synchronization of the ith mode as follows:

$$s + y_{jiR} + y_{kiR} > 0 \tag{51a}$$

$$s(y_{jiR} + y_{kiR}) + 2y_{jiR}y_{kiR} > 0. \tag{51b}$$

In a practical parallel running system, it is to be desired that only the ith mode is stabilized and the other modes (the jth and kth modes) are suppressed. For this purpose we may choose y_j and y_k with comparable values. Now, if we assume that

$$y_j \simeq y_k \tag{52}$$

then the inequality

$$y_{jiR} + y_{kiR} > 0 \tag{53}$$

satisfies simultaneously the two inequalities in (51) approximately. Moreover, (53) derives the following inequalities with (52) also:

$$y_{kjR} + y_{ijR} \simeq y_{ijR} < 0 \tag{54a}$$

$$y_{jkR} + y_{ikR} \simeq y_{ikR} < 0. \tag{54b}$$

Namely both the jth and kth modes become unstable. Consequently (53) is the condition necessary and sufficient for stabilizing only the ith mode under the assumptions of (48) and (52).

For the requirement of more rigorous conditions, numerical calculation is unavoidable. As an example, Fig. 6 exhibits a stability and an instability region of each synchronization mode numerically calculated for Γ_3 when

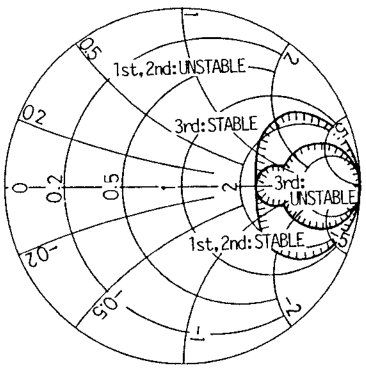


Fig. 6. Stability and instability regions of each synchronization mode numerically derived for Γ_3 of a system with a circularly polarized output power when $s=2$ and $\Gamma_1=\Gamma_2=-0.4$ (on a Smith admittance chart).

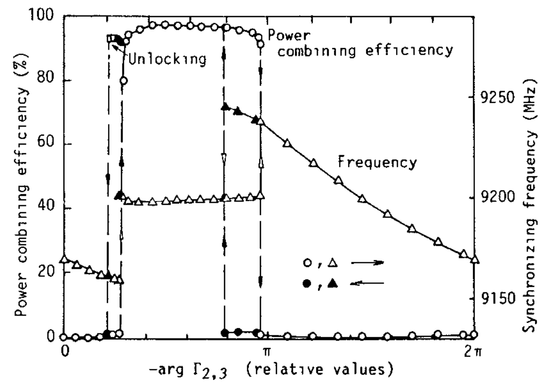


Fig. 7. Measured power-combining efficiency and synchronizing frequency for the power combining system with an output power into a coaxial load.

$s=2$ and

$$\Gamma_1=\Gamma_2=-0.4 \text{ (i.e., } y_1=y_2=2.33). \qquad (55)$$

The figure also indicates that (53) can be regarded as appropriate.

VI. Experimental Results

In order to prove the above-mentioned analytical results, the two different types of power combining (parallel running) systems were constructed of three Gunn oscillators, whose output powers and free-running frequencies were equally adjusted to 68 mW and 9.2 GHz, respectively, and whose external Q's were 70, 71, and 72.

A. System with Combined Output Power to a Coaxial Load

The system shown in Fig. 3 was constructed and tested. Fig. 7 exhibits the measured power-combining efficiencies and synchronizing frequencies as a function of $-\arg\Gamma_c$ for $|\Gamma_c|=0.35$. By inspection of the locus of $|\Gamma_c|=0.35$ illustrated in Fig. 4, it is found that the power-combining synchronization is maintained in a region comparable to region B. Furthermore, the frequency in the synchronism is approximately equal to that in free-running, and the power-combining efficiency is about 97 percent. Outside of this range, the system operates in the three-phase synchronism. In three-phase synchronism, the output power delivered to the coaxial load is very small and below about 0.05 percent in combining efficiency at the middle of the synchronization region. The synchronization frequencies are pulled by Γ_c and their maximum deviation from the free-running frequency is roughly equal to the maximum deviation, 45.4 MHz, of a single oscillator with an external Q of 71 (the average value for the three oscillators) pulled by a load having a reflection coefficient of $|\Gamma_c|=0.35$.

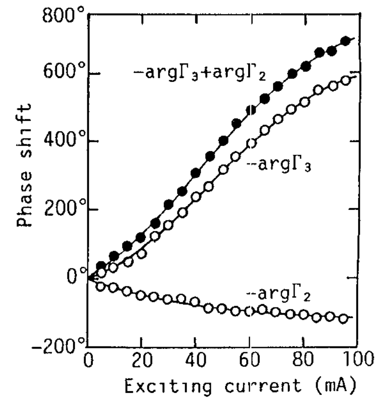


Fig. 8. Phase shift of Γ_2 and Γ_3 versus exciting current for the Faraday rotator (measured).

Hysteretic phenomena also are observed in transitions from one synchronization to the other. This is explained by concurrent stability of both synchronizations.

B. System with Combined Output Power in a Circularly Polarized Wave

An experimental oscillator system was constructed in the manner illustrated in Fig. 5(a). As shown in Fig. 5(b), the load was composed of the arrangement for measurements of the output power in various directions of polarization. The metallic circular aperture and the Faraday rotator employed gave reflections whose magnitudes were 0.37 ($|\Gamma_{2,3}|=0.37$) and whose relative phase angles were determined by the measured phase shifts shown in Fig. 8, respectively. If $\Gamma_1=-0.37$ and initial phase angles of $\Gamma_{2,3}$ (phase angle when the exciting current of the Faraday rotator equals zero) is 123°, we have that $y_1=y_2=2.17$ and $y_3=0.46$ ($\Gamma_2=-0.37$ and $\Gamma_3=0.37$) when the exciting current becomes 27 mA. Then, as described in the previous section, since only the third mode is stabilized, we can obtain a left-hand circularly polarized output. In this case, the circuit of the coaxial arm plays the role of suppressing the in-phase synchronization and holding the three-phase synchronization, though it cannot discriminate the two three-phase synchronizations by itself.

Fig. 9 shows the measured output powers in a direction of polarization parallel to that of Osc. 1 and synchronizing frequencies as a function of the exciting current of the Faraday rotator. As predicted, the third mode is stabilized in a region around the exciting current of 27 mA. Considering that each oscillator in steady state behaves as if it were terminated in the eigenadmittance, we can understand that the synchronizing frequency of the third mode becomes lower because of the phase delay of Γ_3 (i.e., the

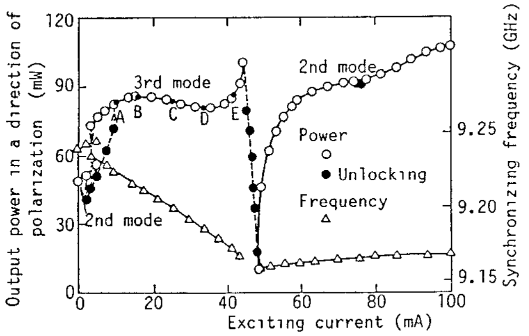


Fig. 9. Measured output power in a direction of polarization and synchronizing frequency versus exciting current of the Faraday rotator.

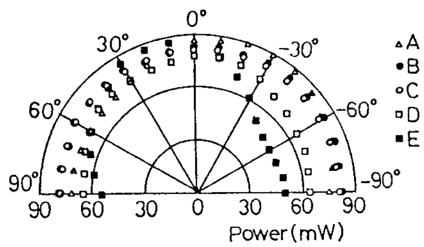


Fig. 10. Measured combined power versus direction of polarization for various values of exciting current.

increase of the susceptance of y_3 in the stability region) with the increase of the exciting current, and oppositely that of the second mode higher because of the phase advance of Γ_2.

Fig. 10 illustrates the measured combined power in various directions of polarization at the exciting currents corresponding to points A, B, C, D, and E dotted in Fig. 9. At points B, C, and D, which exist in the center of the stability region and neighborhood, axial ratios near unity namely, approximate circular polarizations, are obtained, but at points A and E in the edge of the region deterioration of the axial ratios is noticeable. Moreover, the sum of the combined powers in two directions of polarization orthogonal to each other is reduced to about 80 percent of the sum of output powers from the three oscillators. This is presumably caused by the fact that each oscillator runs at a point separated from the optimum operation point or, in other words, it is equivalently terminated in the eigenadmittance different from an optimum admittance (a matched load).

VII. Conclusions

We have made both a theoretical and an experimental investigation concerning the synchronization of three oscillators symmetrically coupled through a six-port magic junction with the aim of power combining, and have suggested two parallel running systems (power combining structures), namely systems with a combined output power in the coaxial load and with a circularly polarized output power combined in the circular waveguide. The one can be easily adjusted and has a simple combining structure, and the other is distinguished by the form of output power directly combined into a circular polarization and electrical switch of the rotatory direction of the circular polarization.

The development of a combiner capable of connecting four or more oscillators and the examination of a power-combining oscillator system utilizing the methods described in this paper would be interesting subjects for further work.

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